



وزارة التربية والتعليم
مكتب العين التعليمي - مدرسة البدع للتعليم الأساسي والثانوي
الصف / الثاني عشر العام

EOT (12-General)

Reveal

هيكل الرياضيات للصف الثاني عشر العام الفصل الدراسي الأول 2025 - 2026م

إعداد الأستاذ /

محمد عبد الحميد الطحاوي

- تم مراعاة الهيكل وترتيب أسئلته
- تم وضع بعض الأسئلة الاسترشادية لكل أسئلة الهيكل
- تم تغيير بعض الأسئلة مع الاحتفاظ بفكرة السؤال لكيلا يحفظ الطالب الأسئلة
- يجب حل هيكل الامتحانات أولاً ثم بعد ذلك حل الامتحانات التجريبية والتي تعتبر وسيلة سريعة للمراجعة

Example 1 Graph a Quadratic Function by Using a Table

Graph $f(x) = x^2 + 2x - 3$. State the domain and range.

Step 1 Analyze the function.

For $f(x) = x^2 + 2x - 3$, $a = 1$, $b = 2$, and $c = -3$.

c is the y -intercept, so the y -intercept is -3 .

Find the axis of symmetry.

$$x = -\frac{b}{2a}$$

Equation of the axis of symmetry

$$= -\frac{2}{2(1)}$$

$a = 1$, $b = 2$

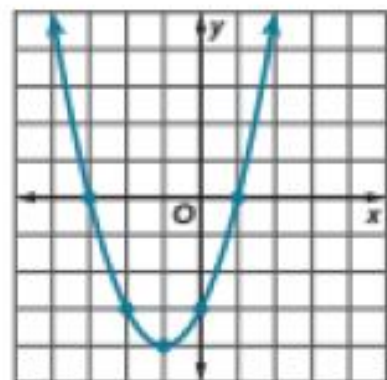
$$= -1$$

Simplify.

The equation of the axis of symmetry is $x = -1$, so the x -coordinate of the vertex is -1 . Because $a > 0$, the vertex is a minimum.

Step 2 Graph the function.

x	$x^2 + 2x - 3$	$(x, f(x))$
-3	$(-3)^2 + 2(-3) - 3$	$(-3, 0)$
-2	$(-2)^2 + 2(-2) - 3$	$(-2, -3)$
-1	$(-1)^2 + 2(-1) - 3$	$(-1, -4)$
0	$(0)^2 + 2(0) - 3$	$(0, -3)$
1	$(1)^2 + 2(1) - 3$	$(1, 0)$



Step 3 Analyze the graph.

The parabola extends to positive and negative infinity, so the domain is all real numbers. The range is $\{y \mid y \geq -4\}$.

Example 1

Graph each function. Then state the domain and range.

1. $f(x) = x^2 + 6x + 8$

2. $f(x) = -x^2 - 2x + 2$

3. $f(x) = 2x^2 - 4x + 3$

4. $f(x) = -2x^2$

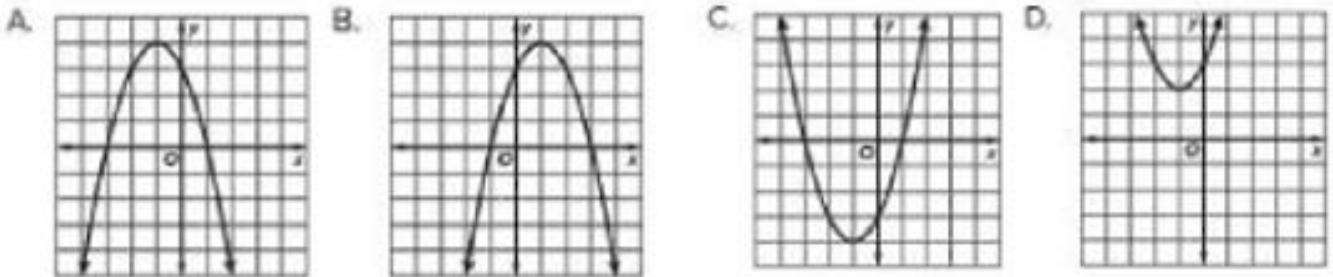
State the domain and range

حدد مجال ومدى الدالة

$$f(x) = x^2 - 4x + 3$$

- A) Domain المجال = All real numbers , Range (المدى) $\{y/y \geq 2\}$
- B) Domain المجال = All real numbers , Range (المدى) $\{y/y \geq -1\}$
- C) Domain المجال = All real numbers , Range (المدى) $\{y/y \leq 2\}$
- D) Domain المجال = All real numbers , Range (المدى) $\{y/y \leq -1\}$

Which in the graph of $f(x) = -x^2 - 2x + 3$



Solve each equation by graphing.

4. $x^2 - 10x + 21 = 0$

5. $4x^2 + 4x + 1 = 0$

6. $x^2 + x - 6 = 0$

7. $x^2 + 2x - 3 = 0$

8. $-x^2 - 6x - 9 = 0$

9. $x^2 - 6x + 5 = 0$

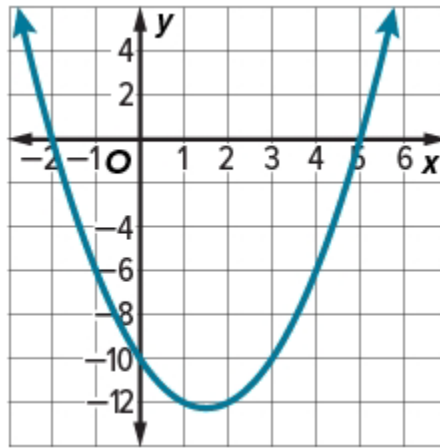
10. $x^2 + 2x + 3 = 0$

11. $x^2 - 3x - 10 = 0$

12. $-x^2 - 8x - 16 = 0$

Use the related graph of equation to determine its solutions

استخدم التمثيل البياني للمعادلة لتحديد حلها



A) $-2, -10$

B) $-2, 5$

C) $0, 5$

D) 5

Example 1 Factor by Using the Distributive PropertySolve $12x^2 - 2x = x$ by factoring. Check your solution.

$$12x^2 - 2x = x$$

Original equation

$$12x^2 - 3x = 0$$

Subtract x from each side.

$$3x(4x) - 3x(1) = 0$$

Factor the GCF.

$$3x(4x - 1) = 0$$

Distributive Property

$$3x = 0 \text{ or } 4x - 1 = 0$$

Zero Product Property

$$x = 0 \quad x = \frac{1}{4}$$

Solve.

Example 2 Factor a TrinomialSolve $x^2 - 6x - 9 = 18$ by factoring. Check your solution.

$$x^2 - 6x - 9 = 18$$

Original equation

$$x^2 - 6x - 27 = 0$$

Subtract 18 from each side.

$$(x + 3)(x - 9) = 0$$

Factor the trinomial.

$$x + 3 = 0 \text{ or } x - 9 = 0$$

Zero Product Property

$$x = -3 \quad x = 9$$

Solve.

Example 4 Factor a Trinomial Where a is Not 1Solve $3x^2 + 5x + 15 = 17$ by factoring. Check your solution.

$$3x^2 + 5x + 15 = 17$$

Original equation

$$3x^2 + 5x - 2 = 0$$

Subtract 17 from each side.

$$(3x - 1)(x + 2) = 0$$

Factor the trinomial.

$$3x - 1 = 0 \text{ or } x + 2 = 0$$

Zero Product Property

$$x = \frac{1}{3} \quad x = -2$$

Solve.

The roots of the quadratic equation $x^2 - 5x + 6 = 0$ are:

A) $x = 1, x = 6$

B) $x = 2, x = 3$

C) $x = -2, x = -3$

D) $x = 0, x = 5$

Q4

Example 4

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Example 4 Complete the Square

Find the value of c that makes $x^2 - 7x + c$ a perfect square. Then write the expression as a perfect square trinomial.

Step 1 Find one half of -7 . $\frac{-7}{2} = -3.5$

Step 2 Square the result from Step 1. $(-3.5)^2 = 12.25$

Step 3 Add the result from Step 2 to $x^2 - 7x$. $x^2 - 7x + 12.25$

The expression $x^2 - 7x + 12.25$ can be written as $(x - 3.5)^2$.

. Find the value of c that makes each trinomial a perfect square.

أوجد قيمة c التي تجعل كل ثلاثي حدود مربعاً كاملاً.

$$x^2 - 6x + c$$

B) -9

B) -6

C) -3

D) 9

Example 5 Solve by Completing the Square

Solve $x^2 + 18x - 4 = 0$ by completing the square.

$$x^2 + 18x - 4 = 0$$

Original equation

$$x^2 + 18x = 4$$

Add 4 to each side.

$$x^2 + 18x + 81 = 4 + 81$$

Add $\left(\frac{b}{2}\right)^2$ to each side.

$$(x + 9)^2 = 85$$

Factor.

$$x + 9 = \pm\sqrt{85}$$

Square Root Property

$$x = -9 \pm \sqrt{85}$$

Subtract 9 from each side.

$$x = -9 + \sqrt{85} \text{ or}$$

Write as two equations.

$$x = -9 - \sqrt{85}$$

$$x \approx 0.22, \text{ or } -18.22$$

Simplify.

The solution set is $\{x \mid x = -9 - \sqrt{85}, -9 + \sqrt{85}\}$.

Solve each equation by completing the square

حل المعادلة بطريقة إكمال المربع

$$x^2 - 4x + 1 = 0$$

A) $x = -2 \pm \sqrt{3}$

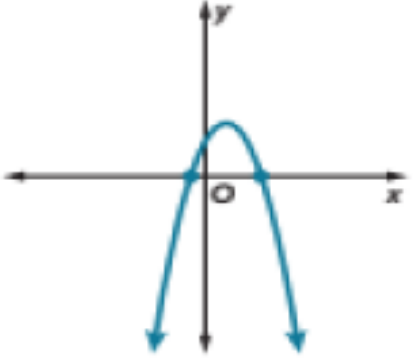
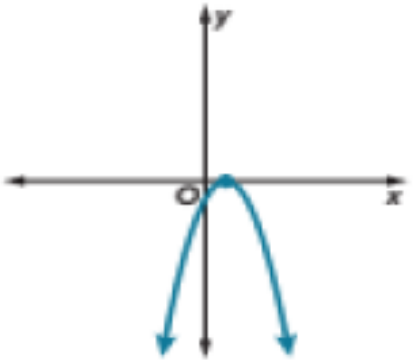
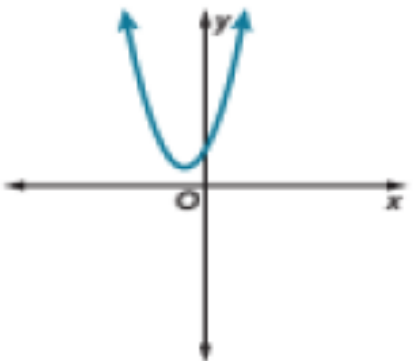
B) $x = 2 \pm \sqrt{5}$

C) $x = 2 \pm \sqrt{3}$

D) $x = 1, 4$

Key Concept • Discriminant

Consider $ax^2 + bx + c = 0$, where a , b , and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is not a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

If the discriminant is negative, the quadratic equation has:

- A) Two real distinct roots
- B) One real repeated root
- C) Two complex roots
- D) No roots

Examples 1 and 2

Graph each inequality.

4. $y \leq x^2 + 4$

5. $y < 2x^2 - 4x - 2$

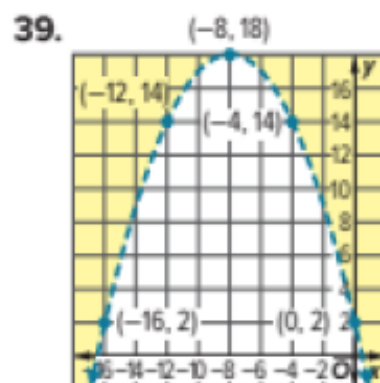
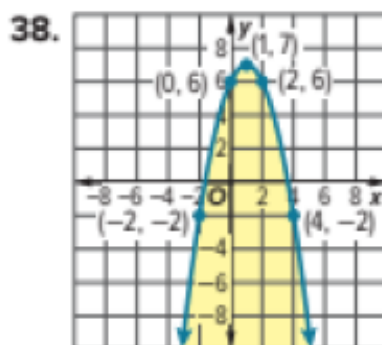
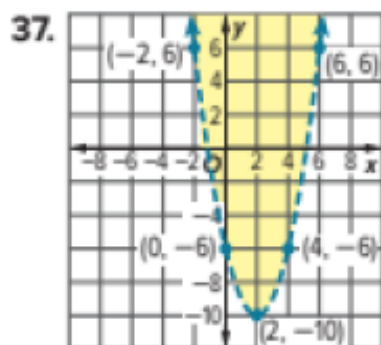
6. $-x^2 + 12x - 36 > y$

7. $y > x^2 + 6x + 7$

8. $y > x^2 - 8x + 17$

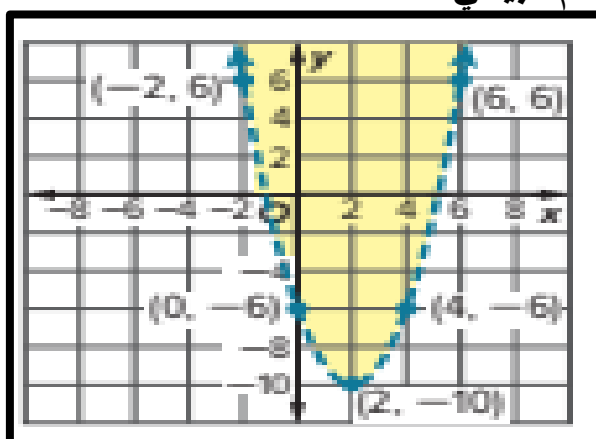
9. $y \geq x^2 + 2x + 2$

Write a quadratic inequality for each graph.



Write a quadratic inequality for the graph

اكتب متباينة تربيعية تمثل الرسم البياني



A) $y > x^2 - 4x - 6$

B) $y > x^2 - 4x - 6$

C) $y \geq x^2 - 4x - 6$

D) $y \leq x^2 - 4x - 6$

Example 3 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

- a. $2x^4 - 3x^3 - 4x^2 - 5x + 6$ degree: 4 leading coefficient: 2
- b. $7x^3 - 2$ degree: 3 leading coefficient: 7
- c. $4x^2 - 2xy + 8y^2$ This is not a polynomial in one variable. There are two variables, x and y .
- d. $x^5 + 12x^4 - 3x^3 + 2x^2 + 8x + 4$ degree: 5 leading coefficient: 1

Check

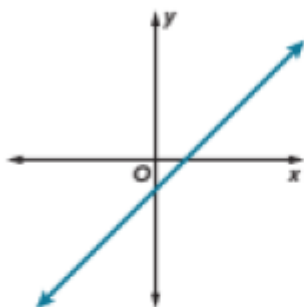
Select the degree and leading coefficient of $11x^3 + 5x^2 - 7x - \frac{6}{x}$.

- A. degree: 3, leading coefficient: 11
- B. degree: 11, leading coefficient: 3
- C. This is not a polynomial in one variable. There are two variables, x and y .
- D. This is not a polynomial in one variable. The term $\frac{6}{x}$ has the variable with an exponent less than 0.

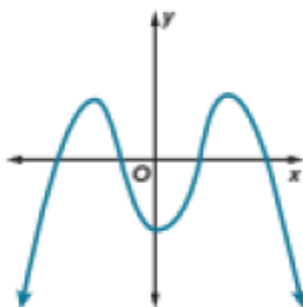
Example 5

Use the graph to state the number of real zeros of the function.

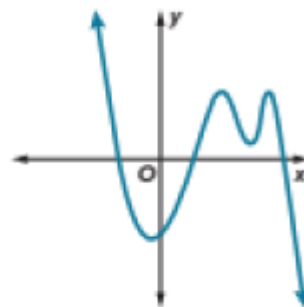
15.



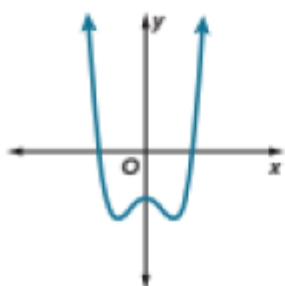
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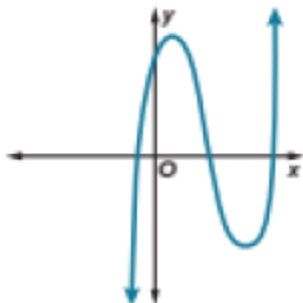
17.



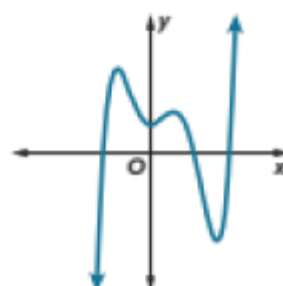
18.



19.

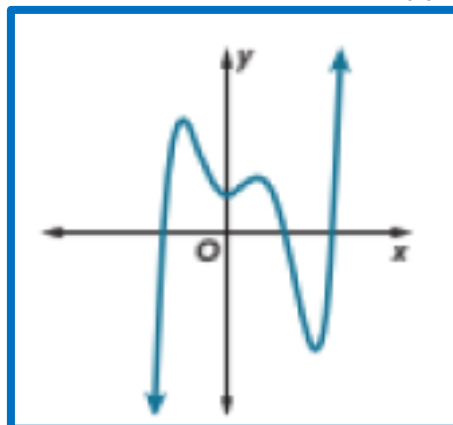


20.



Use the graph to state the number of real zeros of the function

استخدم الرسم البياني لتحديد عدد الجذور الحقيقية للدالة



- A) 2
- B) 5
- C) 3
- D) 0

Example 1 Locate Zeros of a Function

Determine the consecutive integer values of x between which each real zero of $f(x) = x^4 - 2x^3 - x^2 + 1$ is located. Then draw the graph.

Step 1 Make a table.

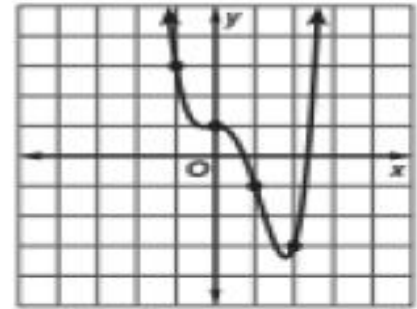
Because $f(x)$ is a fourth-degree polynomial, it will have as many as 4 real zeros or none at all.

x	-2	-1	0	1	2	3	4
$f(x)$	29	3	1	-1	-3	19	113

Using the Location Principle, there are zeros between $x = 0$ and $x = 1$ and between $x = 2$ and $x = 3$.

Step 2 Sketch the graph.

Use the table to sketch the graph and find the locations of the zeros.



Check

Use technology to check the location of the zeros.

Input the function into a graphing calculator to confirm that the function crosses the x -axis between $x = 0$ and $x = 1$ and between $x = 2$ and $x = 3$.

You can find more accurate values of the zeros by using the **zero** feature in the CALC menu to find $x \approx 0.7213$ and $x \approx 2.3486$, which confirms the estimates.

Determine the consecutive integer values of x between which each real zero of each function is located by using a table.

حدد قيم x الصحيحة المتتالية التي تقع بينها كل جذر حقيقي لكل دالة باستخدام جدول

x	-4	-3	-2	-1	0	1
$f(x)$	3	-1	-3	-3	-1	3

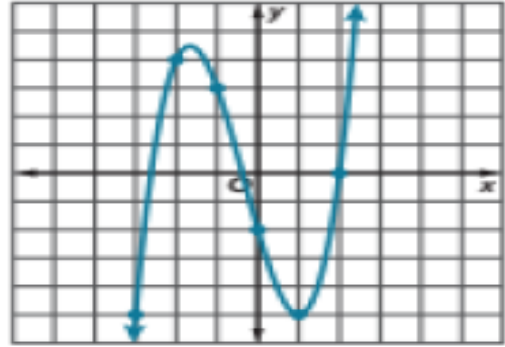
- A) Between $x = -4$, $x = -3$ and $x = 0$, $x = 1$
- B) Between $x = -3$, $x = -2$ and $x = 0$, $x = 1$
- C) Between $x = -2$, $x = -1$
- D) Between $x = 3$, $x = -1$

Example 2 Identify Extrema

Use a table to graph $f(x) = x^3 + x^2 - 5x - 2$. Estimate the x -coordinates at which the relative maxima and relative minima occur.

Step 1 Make a table of values and graph the function.

x	$f(x)$
-4	-30
-3	-5
-2	4
-1	3
0	-2
1	-5
2	0
3	19



Step 2 Estimate the locations of the extrema.

The value of $f(x)$ at $x = -2$ is greater than the surrounding points indicating a maximum *near* $x = -2$.

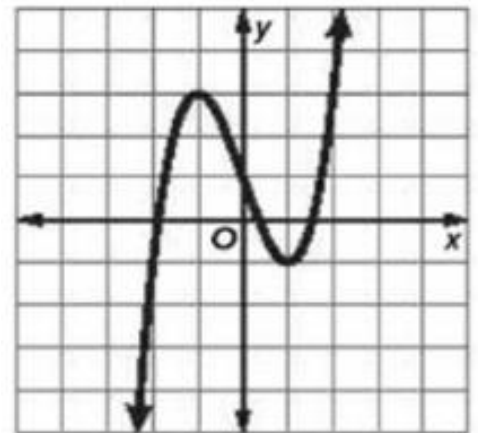
The value of $f(x)$ at $x = 1$ is less than the surrounding points indicating a minimum *near* $x = 1$.

You can use a graphing calculator to find the extrema of a function and confirm your estimates.

Estimate the x -coordinates at which the relative maxima and relative minima occur.

قدّر الإحداثيات x التي تحدث عندها القيم العظمى النسبية والقيم الصغرى النسبية.

- A) max عظمى at $x = -1$
and min صغرى at $x = 1$
- B) max عظمى $x = -1$
and min صغرى at $x = 1$
- C) max عظمى $x = -1$
and min صغرى at $x = 1$
- D) max عظمى at $x = -1$
and min صغرى at $x = 1$



Example 2 Add PolynomialsFind $(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1)$.**Method 1 Add horizontally.**

Group and combine like terms.

$$\begin{aligned}
 &(6x^3 + 7x^2 - 2x + 5) + (x^3 - 4x^2 - 8x + 1) \\
 &= (6x^3 + x^3) + (7x^2 - 4x^2) + (-2x - 8x) + (5 + 1) && \text{Group like terms.} \\
 &= 7x^3 + 3x^2 - 10x + 6 && \text{Combine like terms.}
 \end{aligned}$$

Method 2 Add vertically.

Align like terms vertically and add.

$$\begin{array}{r}
 6x^3 + 7x^2 - 2x + 5 \\
 (+) x^3 - 4x^2 - 8x + 1 \\
 \hline
 7x^3 + 3x^2 - 10x + 6
 \end{array}$$

Align like terms.

Combine like terms.

CheckFind $(2x^3 + 9x^2 + 6x - 3) + (4x^3 - 7x^2 + 5x)$.**Example 3 Subtract Polynomials**Find $(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4)$.**Method 1 Subtract horizontally.**

Group and combine like terms.

$$\begin{aligned}
 &(2x^5 + 11x^4 + 7x - 8) - (5x^4 + 9x^3 - 3x + 4) && \text{Original equation} \\
 &= 2x^5 + 11x^4 + 7x - 8 - 5x^4 - 9x^3 + 3x - 4 && \text{Distribute } -1. \\
 &= 2x^5 + (11x^4 - 5x^4) + (-9x^3) + (7x + 3x) + (-8 - 4) && \text{Group like terms.} \\
 &= 2x^5 + 6x^4 - 9x^3 + 10x - 12 && \text{Combine like terms.}
 \end{aligned}$$

Method 2 Subtract vertically.

Align like terms vertically and subtract.

$$\begin{array}{r}
 2x^5 + 11x^4 + 0x^3 + 7x - 8 \\
 (-) 0x^5 + 5x^4 + 9x^3 - 3x + 4 \\
 \hline
 2x^5 + 6x^4 - 9x^3 + 10x - 12
 \end{array}$$

CheckFind $(8x^2 - 3x + 1) - (5x^3 + 2x^2 - 6x - 9)$.

Find أوجد

$$(5x^2 - 5x + 6) - (2x^2 - 4x + 6)$$

a) $3x^2 - 9x + 12$

b) $7x^2 - 9x - 12$

c) $3x^2 - x + 12$

d) $3x^2 - x$

Q13

(16-26)

97

Examples 4, 5 and 6

Multiply.

17. $(a - 5)^2$

19. $(x - y)(x^2 + 2xy + y^2)$

21. $(x - y)(x + y)(2x + y)$

23. $(r - 2t)(r + 2t)$

25. $(x^3 - 3x^2 + 1)(2x^2 - x + 2)$

16. $x^2(2x + 9)$

18. $(2x - 3)(3x - 5)$

20. $(a + b)(a^3 - 3ab - b^2)$

22. $(a + b)(2a + 3b)(2x - y)$

24. $(3y + 4)(2y - 3)$

26. $(4x^5 + x^3 - 7x^2 + 2)(3x - 1)$

Find أوجد

$$(2x + 3)(4x^2 - 6x + 9)$$

a) $8x^3 - 12x^2 + 18x$

b) $8x^3 - 27$

c) $8x^3 - 24x^2 + 36x + 27$

d) $8x^3 + 27$

Example 1 Divide a Polynomial by a MonomialFind $(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$.

$$(24a^4b^3 + 18a^2b^2 - 30ab^3)(6ab)^{-1}$$

$$= \frac{24a^4b^3 + 18a^2b^2 - 30ab^3}{6ab}$$

Write a fraction.

$$= \frac{24a^4b^3}{6ab} + \frac{18a^2b^2}{6ab} - \frac{30ab^3}{6ab}$$

Sum of quotients

$$= \frac{24}{6}a^{4-1}b^{3-1} + \frac{18}{6}a^{2-1}b^{2-1} - \frac{30}{6}a^{1-1}b^{3-1}$$

Divide.

$$= 4a^3b^2 + 3ab - 5b^2$$

Simplify.

CheckFind $(9x^9y^5 + 21x^4y^4 - 12x^3y^2) \div (3x^2y^2)$.**Example 2** Divide a Polynomial by a BinomialFind $(x^2 - 5x - 36) \div (x + 4)$.

$$\begin{array}{r} x - 9 \\ x + 4 \overline{) x^2 - 5x - 36} \end{array}$$

$$\underline{(-) x^2 + 4x}$$

$$-9x - 36$$

$$\underline{(-) -9x - 36}$$

$$0$$

The quotient is $x - 9$ and the remainder is 0.**Check**Find $\frac{x^2 + 6x - 112}{x - 8}$.

Example 3 Find a Quotient with a Remainder

Find $\frac{3z^3 - 14z^2 - 7z + 3}{z - 5}$.

$$\begin{array}{r} 3z^2 + z - 2 \\ z - 5 \overline{) 3z^3 - 14z^2 - 7z + 3} \\ \underline{(-)3z^3 - 15z^2} \\ z^2 - 7z \\ \underline{(-)z^2 - 5z} \\ -2z + 3 \\ \underline{(-)-2z + 10} \\ -7 \end{array}$$

7) Divide by using long division

استخدم القسمة المطولة في إيجاد ناتج

$$2x^4 + 5x^3 - 2x - 24 \div 2x - 3$$

A) $2x^3 + 8x^2 + 12x + 16$

B) $2x^3 - 8x^2 - 12x - 16$

C) $2x^3 + 11x^2 + 33x + 97 + \frac{267}{2x - 3}$

D) $x^3 + 4x^2 + 6x + 8$

Example 6 Write Expressions in Quadratic Form

Write each expression in quadratic form, if possible.

a. $4x^{20} + 6x^{10} + 15$

Examine the terms with variables to choose the expression equal to u .

$$4x^{20} + 6x^{10} + 15 = (2x^{10})^2 + 3(2x^{10}) + 15 \quad (2x^{10})^2 = 4x^{20}$$

b. $18x^4 + 180x^8 - 28$

If the polynomial is not already in standard form, rewrite it. Then examine the terms with variables to choose the expression equal to u .

$$\begin{aligned} 18x^4 + 180x^8 - 28 &= 180x^8 + 18x^4 - 28 \\ &= 5(6x^4)^2 + 3(6x^4) - 28 \end{aligned} \quad \begin{array}{l} \text{Standard form} \\ \text{of a polynomial} \\ (6x^4)^2 = 36x^8 \end{array}$$

c. $9x^6 - 4x^2 - 12$

Because $x^6 \neq (x^2)^2$, the expression cannot be written in quadratic form.

Check

What is the quadratic form of $10x^4 + 100x^8 - 9$?

Example 7 Solve Equations in Quadratic Form

Solve $8x^4 + 10x^2 - 12 = 0$.

$$8x^4 + 10x^2 - 12 = 0$$

Original equation

$$2(2x^2)^2 + 5(2x^2) - 12 = 0$$

$$2(2x^2)^2 = 8x^4$$

$$2u^2 + 5u - 12 = 0$$

$$\text{Let } u = 2x^2.$$

$$(2u - 3)(u + 4) = 0$$

Factor.

$$u = \frac{3}{2} \text{ or } u = -4$$

Zero Product Property

$$2x^2 = \frac{3}{2} \quad 2x^2 = -4$$

Replace u with $2x^2$.

$$x^2 = \frac{3}{4} \quad x^2 = -2$$

Divide each side by 2.

$$x = \pm \frac{\sqrt{3}}{2} \quad x = \pm i\sqrt{2}$$

Take the square root of each side.

The solutions are $\frac{\sqrt{3}}{2}$, $-\frac{\sqrt{3}}{2}$, $i\sqrt{2}$, and $-i\sqrt{2}$.

Check

What are the solutions of $16x^4 + 24x^2 - 40 = 0$?

$$x = \underline{\hspace{1cm}}?$$

Example 6

Write each expression in quadratic form, if possible.

16. $x^4 + 12x^2 - 8$

17. $-15x^4 + 18x^2 - 4$

18. $8x^6 + 6x^3 + 7$

19. $5x^6 - 2x^2 + 8$

20. $9x^8 - 21x^4 + 12$

21. $16x^{10} + 2x^5 + 6$

Example 7

Solve each equation.

22. $x^4 + 6x^2 + 5 = 0$

23. $x^4 - 3x^2 - 10 = 0$

24. $4x^4 - 14x^2 + 12 = 0$

25. $9x^4 - 27x^2 + 20 = 0$

26. $4x^4 - 5x^2 - 6 = 0$

27. $24x^4 + 14x^2 - 3 = 0$

Solve

$$x^4 - 8x^2 - 9 = 0$$

A) $x = \pm 3, \pm 1$

B) $x = \pm 3$

C) $x = \pm\sqrt{3}$

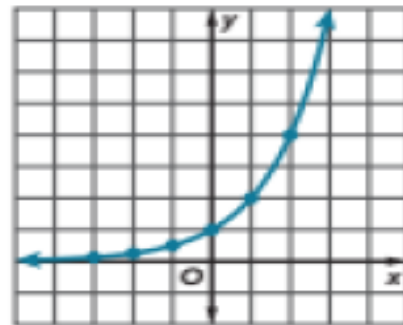
D) $x = -1, 9$

Example 1 Graph Exponential Growth Functions

Graph $f(x) = 2^x$. Find the domain, range, y-intercept, asymptote, and end behavior.

Make a table of values. Then plot the points and sketch the graph.

x	$f(x) = 2^x$
-3	0.125
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8



domain: all real numbers

range: all positive real numbers

y-intercept: (0, 1)

asymptote: $y = 0$

end behavior: As $x \rightarrow -\infty$, $f(x) \rightarrow 0$ and as $x \rightarrow \infty$, $f(x) \rightarrow \infty$.

Example 2 Graph Transformations of Exponential Growth Functions

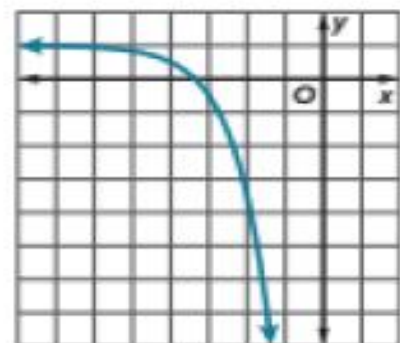
Graph $g(x) = -\frac{1}{2} \cdot 3^{x+4} + 1$.

Transform the graph of $g(x) = 3^x$.

$a = -\frac{1}{2}$; Reflect in the x-axis and compress vertically.

$h = -4$; Translate 4 units left.

$k = 1$; Translate 1 unit up.



Example 6 Graph Exponential Decay Functions

Graph $f(x) = \left(\frac{1}{2}\right)^x$. Find the domain, range, y-intercept, asymptote, and end behavior.

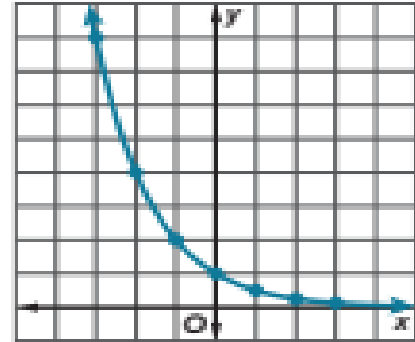
Make a table of values. Then plot the points and sketch the graph.

domain: all real numbers

range: all positive real numbers

y-intercept: (0, 1) asymptote: $y = 0$

end behavior: as $x \rightarrow -\infty$, $f(x) \rightarrow \infty$ and as $x \rightarrow \infty$, $f(x) \rightarrow 0$



Example 7 Graph Transformations of Exponential Decay Functions

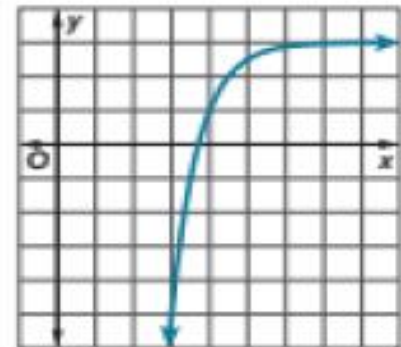
Graph $g(x) = -2\left(\frac{1}{4}\right)^{x-4} + 3$.

$g(x)$ is a transformation of $f(x) = \left(\frac{1}{4}\right)^x$.

$a = -2$; Reflect in the x-axis and stretch vertically.

$h = 4$; Translate 4 units right.

$k = 3$; Translate 3 units up.



Use the graph of $f(x) = 4^x$ to describe the transformation that results in each function. Then sketch the graphs of the functions.

استخدم الرسم البياني للدالة لوصف التحويل الذي ينتج عن كل دالة مما يلي، ثم ارسم الرسوم البيانية الناتجة.

$$f(x) = 4^{x-2}$$

- | | |
|------------------------|---------------------------------|
| a) Shift right 2 units | انتقال بمقدار وحدتين إلى اليمين |
| b) Shift left 2 units | انتقال بمقدار وحدتين إلى اليسار |
| c) Shift up 2 units | انتقال بمقدار وحدتين إلى الأعلى |
| d) Shift down 2 units | انتقال بمقدار وحدتين إلى الأسفل |

□

Example 4 Classify Study Types

UNIFORMS A research team wants to test new football uniform designs and their appeal to young adults. They randomly select 100 young adults to view the different uniforms. The research team observes and records the reactions to the uniforms.

Step 1 What is the purpose of the study?

The purpose is to determine if the new uniforms will be appealing to young adults.

Step 2 Does this situation represent a *survey*, an *experiment*, or an *observational study*?

This is a(n) observational study because the participants are observed without being affected by the study.

Step 3 Identify the sample and population.

The sample is the 100 young adults involved in the study. The population is all young adults.

Check

Match each study subject with the corresponding study type that should be used.

____? A principal wants to determine the favorite after-school activity of his students.

A. observational study

____? A researcher wants to determine whether young adults would be interested in a new line of smartwatches entering the market.

B. census

C. survey

____? A teacher wants to determine whether bright colors affect the test-taking abilities of high school students.

D. experiment

Example 4

Determine whether each situation describes a *survey*, an *experiment*, or an *observational study*. Then identify the sample, and suggest a population from which it may have been selected.

11. An Internet service provider conducts an online study in which customers are randomly selected and asked to provide feedback on their customer service.
12. A research group randomly selects 100 business owners, half of whom started their own businesses, and compares their success.
13. A research group randomly chooses 50 people to participate in a study to determine whether exercising regularly reduces the risk of diabetes in adults.
14. An online video streaming service mails a questionnaire to randomly selected people across the country to determine whether they prefer streaming movies or sports.

One school designed a new logo for it ,
selected 50 students and then monitored
their discussion about the logo
Determine whether the situation calls for

قامت إحدى المدارس بتصميم شعار جديد
للمدرسة ، وتم اختيار 50 طالبا ، ثم رصدت
نقاشهم حول الشعار
حدد ما إذا كان الموقف يستدعي

- A) Survey
- B) Experiment
- C) Observational study
- D) other

استطلاع
تجربة
دراسة مسحية
غير ذلك

Example 3 Identify Biased Questions

FOOD A restaurant owner wants to determine which kinds of meals she should add to her menu: vegetarian options or more traditional meat-based dishes. She releases a survey to her customers asking the following question: *Do you prefer a plain salad or a delicious steak?* Identify any bias in the question.

Step 1 Identify the purpose of the question. The purpose is to determine which dishes are most popular with customers.

Step 2 Identify the bias in the sample. The question provides descriptions of each kind of food, and the description for the steak favors it over the salad.

-
7. Do you think that the workout facility needs a new treadmill and racquetball court?
 8. Which is your favorite type of music, pop, or country?
 9. Are you a member of any after-school clubs?
 10. Don't you agree that employees should pack their lunch?

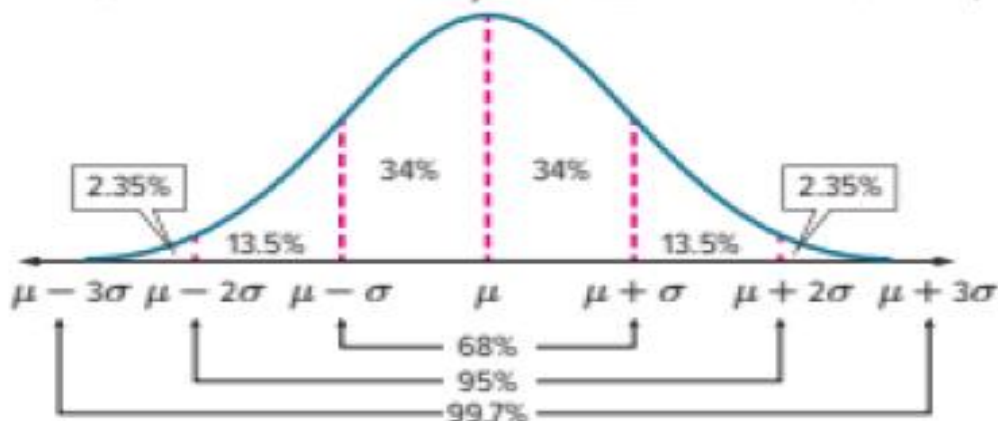
Determine the biased question in the following surveys ?

حدد السؤال المتحيز في الاستطلاعات الآتية ؟

- A) What kind of sport is your favorite?**
ما نوع الرياضة المفضلة لديك
- B) Do you think you need fruits and vegetables daily?**
هل تعتقد احتياجك للفواكه والخضروات يوميا ؟
- C) What subject do you prefer in your studies?**
ما المادة التي تفضلها في دراستك ؟
- D) How many hours do you walk a day?**
كم ساعة تمارس رياضة المشي يوميا ؟

Key Concept • The Empirical Rule

In a normal distribution with mean μ and standard deviation σ ,



- approximately 68% of the data fall within 1σ of the mean,
- approximately 95% of the data fall within 2σ of the mean, and
- approximately 99.7% of the data fall within 3σ of the mean.

When a set of data is *not* approximately normal, it cannot be represented by the Empirical Rule. Skewed data is one example of a set of data that is not approximately normal.



The data are normally distributed, symmetric about the mean, and bell-shaped.



The data are approximately normally distributed. The data can be modeled by the normal distribution.



The data are skewed to the left. A normal curve would not be the best curve to model the distribution.

According to the empirical rule, approximately 95% of the data in a normal distribution lie between which intervals?

وفقاً للقاعدة التجريبية، فإن حوالي 95% من البيانات في التوزيع الطبيعي تقع بين أي من الفترات التالية؟

- $\mu - \sigma$, $\mu + \sigma$
- $\mu - 2\sigma$, $\mu + 2\sigma$
- $\mu - 3\sigma$, $\mu + 3\sigma$
- $\mu - 4\sigma$, $\mu + 4\sigma$

Example 1

Solve each equation by using the Quadratic Formula.

1. $x^2 + 8x + 15 = 0$

2. $x^2 - 18x + 72 = 0$

3. $12x^2 - 22x + 6 = 0$

4. $4x^2 - 6x = -2$

5. $x^2 + 8x + 5 = 0$

6. $-8x^2 + 4x = -5$

12. $x^2 + x - 8 = 0$

13. $8x^2 + 5x - 1 = 0$

Example 4 Use Synthetic DivisionFind $(3x^3 - 2x^2 - 53x - 60) \div (x + 3)$.

Step 1 Write the coefficients of the dividend and write the constant a in the box. Because $x + 3 = x - (-3)$, $a = -3$. Then bring the first coefficient down.

$$\begin{array}{r|rrrr} -3 & 3 & -2 & -53 & -60 \\ & \downarrow & & & \\ & 3 & & & \end{array}$$

Step 2 Multiply by a and write the product. The product of the coefficient and a is $3(-3) = -9$.

$$\begin{array}{r|rrrr} -3 & 3 & -2 & -53 & -60 \\ & & -9 & & \\ \hline & 3 & & & \end{array}$$

Step 3 Add the product and the coefficient.

$$\begin{array}{r|rrrr} -3 & 3 & -2 & -53 & -60 \\ & & -9 & & \\ \hline & 3 & -11 & & \end{array}$$

Step 4 Repeat Steps 2 and 3 until you reach a sum in the last column.

$$\begin{array}{r|rrrr} -3 & 3 & -2 & -53 & -60 \\ & & -9 & 33 & 60 \\ \hline & 3 & -11 & -20 & 0 \end{array}$$

Step 5 Write the quotient. Because the degree of the dividend is 3 and the degree of the divisor is 1, the degree of the quotient is 2. The final sum in the synthetic division is 0, so the remainder is 0.

The quotient is $3x^2 - 11x - 20$.

Example 5 Divisor with a Coefficient Other Than 1

Find $\frac{4x^4 - 37x^2 + 4x + 9}{2x - 1}$.

To use synthetic division, the lead coefficient of the divisor must be 1.

$$\frac{(4x^4 - 37x^2 + 4x + 9) \div 2}{(2x - 1) \div 2}$$

Divide the numerator and denominator by 2.

$$= \frac{2x^4 - \frac{37}{2}x^2 + 2x + \frac{9}{2}}{x - \frac{1}{2}}$$

Simplify the numerator and denominator.

$$x - a = x - \frac{1}{2}, \text{ so } a = \frac{1}{2}.$$

Complete the synthetic division.

$\frac{1}{2}$	2	0	$-\frac{37}{2}$	2	$\frac{9}{2}$
		1	$\frac{1}{2}$	-9	$-\frac{7}{2}$
	2	1	-18	-7	1

The resulting expression is $2x^3 + x^2 - 18x - 7 + \frac{1}{x - \frac{1}{2}}$.
Now simplify the fraction.

$$\frac{1}{x - \frac{1}{2}} = \frac{(1)^2}{\left(x - \frac{1}{2}\right) \cdot 2}$$

Multiply the numerator and denominator by 2.

$$= \frac{2}{2x - 1}$$

Simplify.

The solution is $2x^3 + x^2 - 18x - 7 + \frac{2}{2x - 1}$.

You can check your answer by using long division.

Examples 1-3

Factor completely. If the polynomial is not factorable, write *prime*.

1. $8c^3 - 27d^3$

2. $64x^4 + xy^3$

3. $a^8 - a^2b^6$

4. $x^6y^3 + y^9$

5. $18x^6 + 5y^6$

6. $w^3 - 2y^3$

7. $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$

8. $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$

9. $a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$

10. $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

Example 1

Solve each equation.

1. $25^{2x+3} = 25^{5x-9}$

2. $9^{8x-4} = 81^{3x+6}$

3. $4^x - 5 = 16^{2x-31}$

4. $4^{3x-3} = 8^{4x-4}$

5. $9^{-x+5} = 27^{6x-10}$

6. $125^{3x-4} = 25^{4x+2}$

Example 8 Find Area Under the Standard Normal Curve by Using a Calculator

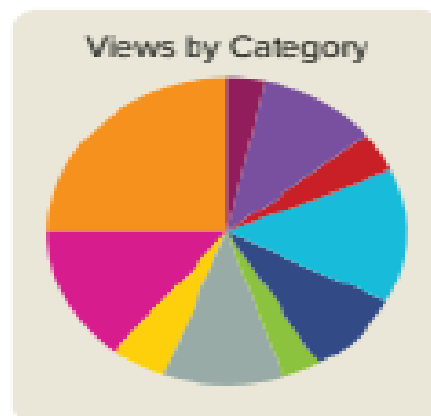
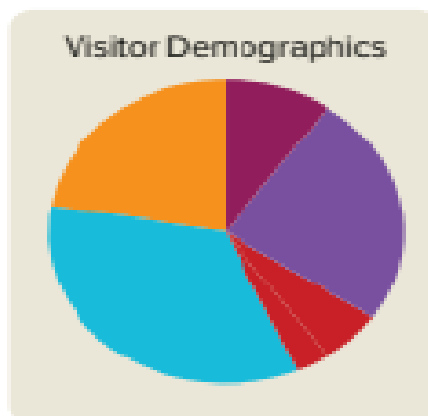
INTERNET TRAFFIC The number of daily hits to a local news Web site is normally distributed with $\mu = 98,452$ hits and $\sigma = 10,325$ hits.

Find the probability that the Web site will get at least 100,000 hits on a given day, $P(X > 100,000)$.

Visits Summary



Visits Summary - Last 30 Days



Step 1 Find the corresponding z -value for $X = 100,000$.

$$z = \frac{X - \mu}{\sigma}$$

Formula for z -value

$$z = \frac{100,000 - 98,452}{10,325}$$

$X = 100,000$, $\mu = 98,452$, and $\sigma = 10,325$

$$z \approx 0.150$$

Simplify.

Step 2 Find the probability.

The area under the curve when $z > 4$ is negligible. So $z = 4$ can be used as an upper bound for finding area. You can use a graphing calculator to find the area between $z = 0.150$ and $z = 4$.

Press **2nd** **[DISTR]** and select **normalcdf**. Enter the interval and press **enter** to display the area.

The area is 0.44. Therefore, the probability of the Web site getting at least 100,000 hits in one day is 44.0%.

```
normalcdf(0.150,
4)
.4403596819
```

Example 8

14. **TESTING** The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 12.3.
- What percent of the scores are between 70 and 80?
 - What percent of the scores are over 115?
 - If 75 people take the test, how many would you expect to score lower than 75?